

One Sided Access in Two-Sided Markets*

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Abstract

In this paper, I analyze the incentives of a monopolistic platform to open its infrastructure to an entrant on the buyer side of the market. The platform exerts an externality on the entrant when it chooses the seller price, because the entrant values the presence of sellers on the platform. The entrant pays an access charge to the platform for each transaction. Therefore, the entrant exerts an externality on the platform when it chooses its price because it impacts the revenues that the platform obtains from opening its infrastructure. If buyer and seller demands are linear and identical, and if the degree of product differentiation is low, I show that the platform's price structure is biased in favor of sellers. If the degree of product differentiation is high, the price structure is biased in favor of sellers only for high values of the access charge.

Keywords: two-sided markets, access, entry.

JEL Codes: E42; L1; O33.

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1 Introduction

In several network industries (e.g., the payment card industry, the video game industry), a dominant platform or a duopoly of incumbent platforms organize the interactions between two distinct groups of users. These markets are often referred to as "two-sided" markets, following the works of Rochet and Tirole (2003), Caillaud & Julien (2003) and Armstrong (2006). In such markets, because of network effects, smaller firms are often reluctant to build their own platform when they innovate because competition with incumbent platforms is unlikely to yield enough profits to cover their fixed costs. One solution for these firms is therefore to use the incumbent's infrastructure to market new products. However, an incumbent may refuse to open its infrastructure to an entrant, unless this strategy generates enough profits. In Antitrust, such strategies have been discussed in the framework of the foreclosure doctrine (see *Terminal Road Association versus U.S.* (1912)). While the foreclosure doctrine has been extensively analyzed in vertically related markets or in the telecommunications industry¹, no paper has studied whether payment platforms have incentives to foreclose access to entrants. This paper aims at providing a perspective on this issue.

Currently, the issue of access to payment platforms seems a major policy concern for the regulators of the payments industry. Indeed, non-banks try to enter the market by using the existing platforms of banks to market new products, either on the buyer side of the market or on the seller side. On the buyer side, several non-banks have designed payment solutions which rely on the existing payment instruments offered by banks to settle transactions with sellers (e.g., mobile payment solutions that rely on the payment card, m wallets). On the seller side, non-banks have also tried to offer innovations that enable merchants to accept payment cards through other access channels (e.g., Square for mobile payments).

In this context, the theory of one-way access, which has been designed for the telecommunications industry (see Vogelsang, 2003, for a survey), has to be adapted to account for the specific nature of two-sided markets.² Indeed, in two-sided markets, the decision of a platform to open its infrastructure on one side of the market may be affected by the revenues that the platform can obtain from the other side. Conversely, an entrant's decision to enter on one side of the market may depend on the platform's prices for both sides.

¹For instance, in the European Union, telecommunication operators with significant market power are required to charge cost-based access prices (see the interconnexion directive of 1997).

²My paper also presents some similarities with the literature on two-way access and termination charges (see Hermalin and Katz, 2011 or Cambini and Valletti, 2008), one side of the market being the senders of messages and the other side being the receivers of messages. See section 2 for a survey of the literature.

To address this issue, I build a generic model of entry in a two-sided market, which is based on the framework of Rochet and Tirole (2003). A monopolistic platform has to decide whether to provide access to its infrastructure to an entrant on one side of the market, say the buyer side. If the platform opens its infrastructure, the entrant pays a per transaction access fee to the platform, which is not restricted to be positive. As the entrant incurs fixed entry costs, the platform may use the access fee to foreclose entry. If the platform opens its infrastructure, the entrant chooses how to price transactions for buyers, and the platform chooses how to price transactions for buyers and sellers, respectively.

In my setting, entry has two effects on the platform's profits: a business stealing effect on the buyer side and an income effect on the seller side. The income effect arises because the platform earns revenues from the seller price and from the access fee when the entrant's consumers make transactions with sellers. In order to isolate the income effect from the business stealing effect, I first study a benchmark, in which the entrant enters on a separate market (e.g., mobile payments on the Internet). If there is entry, firms exert externalities on each other at the price competition stage. There are two kinds of externalities in my framework. First, the platform exerts an externality on the entrant through the choice of the seller price, because seller demand impacts the volume of transactions made by the entrant's consumers. Second, the entrant exerts an externality on the platform when it chooses its price, because the platform earns revenues from the entrant's consumers through the access charge and the seller fee. The platform chooses prices for buyers and sellers that balance the profits earned on the initial market and on the new market. If seller and buyer demands are linear and identical and if there is entry, I show that for high values of the access charge, the profit-maximizing price structure of the platform is biased in favor of sellers, because it increases the transaction volume on the new market. By contrast, for low values of the access charge, the price structure is biased in favor of buyers. The entrant's decision to enter the buyer market depends on the level of the access charge. The access charge impacts the entrant's profits through a direct and an indirect effect, which depends on the price that the platform chooses for sellers. I show that the entrant's profits may increase with the access charge because the entrant values the presence of sellers on the platform. Conversely, the platform's profits can decrease with the access charge, because the platform earns revenues from the entrant's transactions. I am able to provide sufficient conditions such that the platform accommodates entry in a general setting, and I determine the profit-maximizing access charge if demands are linear.

To analyze the business stealing effect, I assume that if the entrant enters the market, it competes

in prices with the platform by offering differentiated services to buyers. I show that the business stealing effect reinforces the platform's incentives to bias the price structure in favor of sellers. Indeed, the platform has an incentive to increase its price for buyers because it earns revenues from the entrant's consumers. These revenues are all the more sensitive to the buyer price since the degree of product differentiation is low. Compared to the benchmark, a higher platform price for buyers has a positive impact on the entrant's profits, because it increases the entrant's demand. Since the platform's price for buyers increases with the access charge, a higher access charge can increase the entrant's profits. The magnitude of this effect is all the more important since the degree of product differentiation is low. Consequently, I find that the entrant's profits increase with the access charge if the degree of product differentiation is low. By contrast, if the degree of product differentiation is high, the entrant's profits decrease with the access charge. The platform's profits increase with the access charge if the sensitivity of the entrant's demand is low. If the prices chosen by the platform and by the entrant for buyers are identical, and if the size of the market is fixed, I show that, if the platform accommodates entry, the entrant makes zero profit.

The remainder of the article is organized as follows. In section 2, I review the literature that is related to my study. In section 3, I present the model and the assumptions. In section 4, I analyze a benchmark case, in which the entrant enters on a separate market and does not compete with the platform. In section 5, I consider the general case, in which the entrant competes with the platform on the buyer side of the market. In section 6, I assume that the entrants are perfectly competitive and operate on a separate market. Finally, I conclude. All proofs are in the Appendix.

2 Literature review

This paper is related to three strands of the literature: the literature on market foreclosure, the literature on interconnection in telecommunications networks and the literature on entry in two-sided markets.

The literature on market foreclosure studies whether a vertically integrated firm has an incentive to foreclose the downstream market to its rivals, either by raising its rival's cost (see Salop and Scheffman, 1987, or Vickers, 1995) or by degrading the quality of service offered to the entrant (see Economides, 1998). In my paper, the entrant needs the incumbent to interconnect its consumers to the platform's sellers, and, therefore, the platform may use the access charge to restrict access to its captive base of sellers.

My paper is also related to the literature on interconnection in telecommunications networks (See Laffont and Tirole, 2000, Armstrong, 2002 and De Bijl and Peitz, 2002, for surveys of this literature). Many telecommunications service providers own partial networks and rely on the incumbent's infrastructure to offer services to their customers. As surveyed by Vogelsang (2003), the literature on one-way access studies whether dominating network providers have incentives to give competitors access to an infrastructure that is hard to duplicate. My paper extends the analysis of competitive bottlenecks in telecommunications infrastructure to the case of two-sided payment platforms. An important question for Central Banks and competition authorities is whether incumbent payment networks have incentives to grant access to payment service providers or non-banks (see the CPSS Report on innovation in retail payment systems, 2012). These new players often decide to enter the market on one side by offering different access channels to the existing payments infrastructure.

My paper also presents some similarities with the literature on two-way access and termination charges. Mobile networks can also be analyzed as two-sided markets, with senders of messages on one side and receivers on the other side (see Katz and Hermalin, 2011). In this literature, most papers assume that networks compete in two-part tariffs and that networks charge each other a reciprocal access charge when consumers send messages. My paper makes several assumptions that depart from this literature and that apply more specifically to the case of the retail payments industry. First, networks do not compete to attract users and all users multihome. This assumption is consistent with the fact that in developed countries, virtually all consumers are equipped with a payment card and a mobile phone. Second, networks are asymmetric, because one network has attracted senders and receivers (buyers and sellers) whereas the other network has only attracted senders.

The literature on entry in two-sided markets is scarce. Farhi and Hagiu (2008) analyze the strategic interactions between an incumbent platform and an entrant in a general framework. They propose an adaptation of the typology offered by Fudenberg and Tirole (1984) to the framework of two-sided markets. They show that strategic interactions between two-sided platforms depend not only on whether their decision variables are strategic complements or substitute as for one-sided firms, but also on whether or not platforms subsidize one side of the market in equilibrium. Dewenter and Roesch (2012) consider platforms which compete in quantities on two interrelated markets. They analyze the impact of the number of firms on quantities and prices. They show that a free entry equilibrium yields to excessive entry if network effects are large. Tregouët (2012)

studies whether a platform has an incentive to integrate vertically with sellers during the launch phase of a new platform. He shows that integration is more profitable when the trade surplus is shared equally between the two sides of the market.

3 The Model

In this section, I build a generic model of access to a two-sided platform, which can be applied in particular to the retail payments industry.

A monopolistic payment platform (PF) provides an intermediation service to a group of buyers (B) and a group of sellers (S). The marginal cost of serving group B is c_B and the marginal cost of serving group S is c_S . The platform's total marginal cost is $c = c_B + c_S$. The prices charged to buyers and sellers are p_B and p_S , respectively. The platform's profits are denoted by π^{PF} .

The platform can decide to open its infrastructure to a monopolistic entrant (E), that offers access services to buyers.³ The entrant charges buyers with a service fee p_E for each transaction and pays an access charge a per transaction to the incumbent platform.⁴ It also incurs a marginal cost c_E to provide the service to its consumers. For example, a payment card platform can decide to allow mobile network operators to use its infrastructure to offer payment services to their consumers. If there is entry, the entrant and the platform compete in prices on the buyer side of the market, unless the entrant serves a separate market, which is the case that I study as a benchmark. The entrant decides to enter the market if its gross profits π^E exceed the fixed entry cost ϕ . All buyers and sellers multihome, that is, they are equipped with the payment solution offered by the platform (e.g., the payment card) and with the access technology offered by the entrant (e.g., the mobile phone).

On the buyer side of the market, consumers have to decide which channel to use to access the platform's services. The access channel offered by the platform and the entrant are not perfectly homogenous. Given the prices charged by the platform and the entrant, there are $D_B^{PF}(p_B, p_E)$ buyers who prefer to access the platform directly and $D_B^E(p_B, p_E)$ buyers who prefer to access the

³We would obtain similar results if a platform decided to open its infrastructure to sellers, that need the platform to make transactions with buyers. For example, a merchant that owns a platform to do business on the Internet could decide to allow other merchants to market products on its platform (e.g., Amazon).

⁴The letter a for the access charge is used to make comparisons with the literature on interconnection in telecommunications networks. However, it should not be confused with the letter a that is generally used for interchange fees in the literature on payment systems.

platform via the solution offered by the entrant. The total consumer demand is

$$D_B(p_B, p_E) = D_B^{PF}(p_B, p_E) + D_B^E(p_B, p_E).$$

On the seller side of the market, there are $D_S(p_S)$ sellers who wish to use the platform to make a transaction. A transaction is intermediated by the platform only if the buyer and the seller agree for it, given the prices they pay for it. Therefore, the total volume of transactions is $D_S(p_S)D_B(p_B, p_E)$.

Finally, note that I do not model the price p that sellers charge to buyers on the retail market (as in Rochet and Tirole, 2003, 2006). This implies that the prices charged by the platform and the entrant do not influence the buyer's decision to buy the good that is offered by each merchant. While this assumption may seem exagurate, it is a useful approximation to understand a monopolistic platform's incentives to open its infrastructure to an entrant. I will relax this assumption in the extension section.

I make the following assumptions, which are satisfied in particular if quasi-demands are linear:

Assumption 1 (Demand functions): Demand functions are twice differentiable, decreasing and concave. A firm's demand increases with the price that is chosen by its competitor, that is, we have $\partial D_B^{PF}/\partial p_E \geq 0$ and $\partial D_B^E/\partial p_B \geq 0$. Furthermore, I assume that $\partial^2 D_B^{PF}/\partial p_B \partial p_E = \partial^2 D_B^E/\partial p_B \partial p_E = 0$.

Assumption 2 (Concavity): The platform's profits are concave in its prices, holding the entrant's price as constant and the entrant's profits are concave in its price, holding the platform's prices as constant. The assumption that the platform's profits are concave in the entrant's price implies that

$$\Delta_0 = (\partial^2 \pi^{PF}/\partial p_B^2)(\partial^2 \pi^{PF}/\partial p_S^2) - (\partial^2 \pi^{PF}/\partial p_S \partial p_B)^2 > 0.$$

Assumption 3 (Strategic complementarity): If the platform and the entrant compete in prices on the same market, prices are strategic complements; that is, we have $dp_B^{BR}/dp_E > 0$ and $\partial p_E^{BR}/\partial p_B > 0$, where p_B^{BR} denotes the platform's best-response to the entrant's price and p_E^{BR} denotes the entrant's best-response to the platform's prices. A sufficient condition for prices to be strategic complements is that

$$\Delta_1 = \frac{\partial^2 \pi^{PF}}{\partial p_S^2} \frac{\partial^2 \pi^{PF}}{\partial p_B \partial p_E} - \frac{\partial^2 \pi^{PF}}{\partial p_B \partial p_S} \frac{\partial^2 \pi^{PF}}{\partial p_E \partial p_S} < 0.$$

Assumption 4: I assume that

$$(\partial^2 \pi^E / \partial p_E^2) \Delta_0 - (\partial^2 \pi^E / \partial p_E \partial p_B) \Delta_1 < 0.$$

This assumption ensures that the determinant of the matrix of the cross-derivatives $(\partial^2 \pi^k / \partial p_i \partial p_j)$ for $k = PF, E$, and $(i, j) \in \{B, E, S\}$ is negative. This assumption is necessary to demonstrate comparative statics results about the variation of the equilibrium prices with the access fee.

I am interested in understanding the impact of the access charge on firms' profits and entry. Therefore, the timing of the game that I study is as follows:

1. The platform chooses whether to open its infrastructure to the entrant. If the platform opens its infrastructure, it decides on the level of the access charge a that is paid per transaction by the entrant.
2. The entrant decides whether or not to enter the market.
3. If the platform has not opened its infrastructure at the first stage or if the entrant has not entered at stage 2, the platform chooses the monopolistic prices p_B^m and p_S^m . If the platform has opened its infrastructure and if the entrant has entered the market, both firms compete in prices on the buyer side and the platform chooses how to price transactions on the seller side.

I look for the subgame perfect equilibrium of the game and solve it through backward induction.

4 A benchmark: no competition

In this section, I analyze a special case in which the entrant does not compete with the platform on the buyer side (i.e., there is no business stealing). Therefore, if there is entry, the entrant sells its services on a separate market. This implies that $D_B^{PF}(p_B, p_E) \equiv D_B^{PF}(p_B)$ and $D_B^E(p_B, p_E) \equiv D_B^E(p_E)$. For instance, in the retail payments industry, one could consider an entrant that develops a payment solution for consumers to pay on the Internet which relies on the platform's network. The payment instrument offered by the platform cannot be used to pay on the Internet and vice versa.

4.1 Stage 3: prices

If the entrant has entered at stage 2, the platform and the entrant choose the prices p_B , p_S and p_E that maximize their profits, respectively; that is,

$$\pi^{PF} = D_S(p_S)(D_B^{PF}(p_B)(p_B + p_S - c) + D_B^E(p_E)(a + p_S - c_S)), \quad (1)$$

and

$$\pi^E = D_S(p_S)D_B^E(p_E)(p_E - a - c_E). \quad (2)$$

Though the platform and the entrant do not compete, they exert externalities on each other when they choose their prices. On the one hand, the entrant's profits depend on the platform's price for sellers because it impacts the transaction volume on the new market ($D_B^E(p_E)D_S(p_S)$). On the other hand, the platform's profits depend on the entrant's price. Indeed, the platform earns profits on the new market, because it receives the seller price and the access charge for each transaction that is made by the entrant's consumers.

The best-response functions of the platform and the entrant are denoted by $p_B^{BR}(p_E)$, $p_S^{BR}(p_E)$, and $p_E^{BR}(p_B, p_S)$, respectively. I start by determining the platform's best-responses to the entrant's price. Solving for the first-order conditions of platform's profit-maximization yields

$$D_S(p_S) \left(\frac{dD_B^{PF}}{dp_B}(p_B + p_S - c) + D_B^{PF} \right) = 0, \quad (3)$$

and

$$D_S(p_S)(D_B^{PF}(p_B) + D_B^E(p_E)) + \frac{dD_S}{dp_S}(D_B^{PF}(p_B)(p_B + p_S - c) + D_B^E(p_E)(a + p_S - c_S)) = 0. \quad (4)$$

Eq.(3) and (4) can be rearranged as

$$\frac{p_B + p_S - c}{p_B} = \frac{1}{\eta_B^{PF}}, \quad (5)$$

and

$$\frac{p_B}{p_S} = \frac{\eta_B^{PF}}{\eta_S} \left(\frac{D_B^{PF} + D_B^E}{D_B^{PF}} \right) + \frac{D_B^E D_S(a + p_S - c_S)}{D_B^{PF}}, \quad (6)$$

where η_B^{PF} is the elasticity of the platform's consumers quasi-demand and η_S is the elasticity of

seller quasi-demand.⁵ Eq.(5) and (6) differ from the usual monopoly pricing equations of Rochet and Tirole (2003), because the platform earns profits from the new market. Therefore, from (4), the platform's price for sellers depends on the price that is chosen by the entrant. If the platform increases its price for sellers, it earns marginal benefits on the new market (equal to $D_B^E(p_E)D_S(p_S)$) and incurs marginal losses (if $a + p_S - c_S > 0$) because the transaction volume decreases by $(dD_S/dp_S)D_B^E(p_E)$. From (3), it is noteworthy that the platform's price for buyers only depends on the entrant's price through the seller price.

Lemma 1 analyzes the impact of the entrants' price on the platform's best-responses according to the level of the access charge.

Lemma 1 *If the access charge is lower than the platform's net benefit of serving the entrant's consumers (that is, if $a < p_S^{BR} - c_S$), the platform's best-response on the seller side decreases with the entrant's price, whereas the platform's best-response on the buyer side increases with the entrant's price.*

When the entrant's price increases, the entrant's demand decreases, which has two consequences on the platform's incentives to increase its price for sellers. On the one hand, a higher entrant's price reduces the platform's marginal benefits of increasing its price for sellers. The reduction in the marginal benefits of the platform is equal to $D_S(p_S)(dD_B^E/dp_E)$. On the other hand, the platform's marginal costs of increasing its price decrease (by $(dD_S(p_S)/dp_S)(dD_B^E/dp_E)(a + p_S - c_S)$), provided that the platform earns profits from serving the entrant's consumers. If the platform incurs losses from serving the entrant's consumers, the platform has an incentive to lower its price for sellers if the entrant's price increases. Finally, the platform's best-response on the buyer side is impacted indirectly by the entrant's price variation through the seller price. Since a lower seller price increases the buyer price, the platform has an incentive to increase the buyer price when the entrant's price increases.

It is also interesting to analyze whether entry on the buyer side changes the structure of the prices charged by a monopolistic platform.⁶ In Lemma 2, I compare the platform's best-responses in case of entry to the prices that are chosen by a monopolistic platform.

⁵In the two-sided markets literature, the quasi-demand refers to the probability that a consumer on one side of the market wishes to use the platform to make a transaction, holding the price paid by the other side as constant.

⁶In the two-sided markets literature, the price structure refers to the difference between the buyer price and the seller price, or the ratio of prices between the buyer price and the seller price.

Lemma 2 *If the access charge is lower than the net costs of serving the platform's consumers under monopoly (that is, if $a < p_B^m - c_B$), if there is entry on the buyer side, the price for sellers increases and the price for buyers decreases; that is, we have $p_S^{BR}(p_E) \geq p_S^m$ and $p_B^{BR}(p_E) \leq p_B^m$. Otherwise, we have $p_S^{BR}(p_E) \leq p_S^m$ and $p_B^{BR}(p_E) \geq p_B^m$.*

If there is entry on the market, the platform obtains revenues from its consumers and from the entrant's consumers. If the access charge is high, the platform has an incentive to increase the transaction volume on the new market by lowering the price for sellers. Proposition 1 demonstrates that the platform lowers the price for sellers if the margin per consumer is higher on the new market than on the initial market. By contrast, if the revenues from the new market are lower than on the initial market, the platform has an incentive to increase its price for sellers and to lower its price for buyers.

I proceed by analyzing the entrant's pricing strategy. Solving for the first-order condition of entrant's profit-maximization yields

$$D_B^E + \frac{dD_B^E}{dp_E}(p_E - a - c_E) = 0. \quad (7)$$

Because the entrant's price does not depend on the platform's prices, the entrant's best-response is to choose the monopoly price on the new market at the equilibrium of stage 3. I denote the entrant's price by $p_E^m(a)$. The platform's best-response is to choose the prices $p_B^{BR}(p_E^m(a))$ and $p_S^{BR}(p_E^m(a))$ for buyers and sellers, respectively. I denote the vector of the prices chosen at the equilibrium of stage 3 by $P^* = \{p_B^e(a), p_S^e(a), p_E^e(a)\}$. Proposition 1 compares the equilibrium prices when there is entry to the prices charged by a monopolistic platform.

Proposition 1 *If $a < p_B^m - c_B$, if there is entry on the buyer side, the price for sellers increases and the price for buyers decreases; that is, we have $p_S^e(a) \geq p_S^m$ and $p_B^e(a) \leq p_B^m$. Otherwise, we have $p_S^e(a) \leq p_S^m$ and $p_B^e(a) \geq p_B^m$.*

Proof. From Lemma 1 at the equilibrium of the subgame. ■

An example with linear demands: Let $D_B^{PF} = \max\{1 - \gamma_B p_B, 0\}$, $D_S^{PF} = \max\{1 - \gamma_S p_S, 0\}$, and $D_E^E = \{1 - \delta_E p_E, 0\}$, where $\gamma_i > 0$ for $i = B, S$ and $\delta_E > 0$. Assume that $c = c_E = 0$ and that $\gamma_S = \gamma_B = \delta_E = 1$. In this example, the Nash equilibrium of the subgame is $p_B^e(a) = (6 - 2a - \sqrt{18 - 24a + 10a^2})/6$, $p_E^e(a) = (1 + a)/2$, and $p_S^e(a) = (-3 + 2a + \sqrt{18 - 24a + 10a^2})/3$

for all $a \in [-1, 1]$.⁷ If the access charge is $a \geq 1$, the prices for buyers and for sellers are equal to the monopoly prices $1/3$ and $1/3$, respectively, and the entrant does not enter because $D_B^E = 0$.

It is interesting to analyze the structure of the prices charged by the platform at the equilibrium of stage 3 when there is entry. If the platform is a monopolist, and if buyer and seller demands are linear and identical, the platform chooses identical prices for buyers and sellers (see Rochet and Tirole, 2003). In Proposition 2, I demonstrate that, in case of entry, if buyer and seller demands are linear and identical, the platform's price structure is biased in favor of sellers for high values of the access charge, in order to increase the transaction volume on the new market.

Proposition 2 *Assume that $c = c_E = 0$, $D_B^{PF} = \max\{1 - \gamma_B p_B, 0\}$, $D_S^{PF} = \max\{1 - \gamma_S p_S, 0\}$, and $D_B^E = \{1 - \delta_E p_E, 0\}$.*

- i) If $\gamma_B \geq 7\gamma_S$, we have $p_B^e(a) \leq p_S^e(a)$ for all $a \in [-1/\delta_E, 1/\delta_E]$.*
- ii) If $\gamma_B < 7\gamma_S$, the sign of $p_B^e(a) - p_S^e(a)$ depends on the value of the access charge and on the relative demand elasticities on each market. In particular, if $\gamma_B = \gamma_S$ and $a \leq 1/3\gamma_B$, $p_B^e(a) \leq p_S^e(a)$. If $a > 1/3\gamma_B$, $p_B^e(a) > p_S^e(a)$.*

In two-sided markets, entry on one side of the market may impact the prices chosen on the other side. Since the platform earns profits on the entrant's market, it has an incentive to choose a price for sellers that balances the profits earned from the initial market and from the new market. Proposition 2 demonstrates that if the access charge is high, the platform increases the transaction volume on the new market by lowering the seller price when buyer and seller demands are linear and identical. Interestingly, even if the platform does not compete with the entrant to attract buyers, the price for buyers is impacted by entry because the price structure is biased in favor of sellers for high values of the access charge.

Another interesting situation is the case where the elasticities are the same on the new market and on the buyer side, that is, if $\gamma_B = \delta_E$. Proposition 3 analyzes the impact of entry on the price structure if $\gamma_B = \delta_E$ in the case of linear demands.

If $\delta_E \leq 6\gamma_S/7$, $p_S^e(a) \leq p_B^e(a)$ for all $a \in [-1/\delta_E, 1/\delta_E]$. If $6\gamma_S/7 \leq \delta_E < \delta_E^*$, $p_S^e(a) > p_B^e(a)$ for low values of a and then $p_S^e(a) < p_B^e(a)$. If δ_E intermediary, $p_S^e(a) > p_B^e(a)$ for all

⁷For all $a \in [0, 1]$, we have $18 - 24a + 10a^2 > 0$. Note that in this case, the platform's best-responses are not uniquely defined by the first-order conditions. However, I am able to select the prices that maximize the platform's profit.

values of $a \in [-1/\delta_E, 1/\delta_E]$. If Delta large, $p_S^e(a) > p_B^e(a)$ for small and large values of a and $p_S^e(a) < p_B^e(a)$ for intermediary values of a .

Finally, in Lemma 3, I explain how the access charge impacts the platform's prices and the entrant's price.

Lemma 3 *The entrant's price increases with the access charge. If the sensitivity of the entrant's demand is low, the seller price decreases with the access charge. If the sensitivity of seller demand and the sensitivity of the entrant's demand are high, the seller price may decrease with the access charge for low values of the access charge and then increase with the access charge.*

The entrant's price increases with the access charge because it increases its marginal cost. By contrast, the seller price varies non-monotonically with the access charge if the sensitivity of seller demand and the sensitivity of the entrant's demand are high. Two effects are at play: a direct effect and an indirect effect. First, if the access charge increases, the platform has an incentive to decrease its price for sellers because the platform earns revenues from the entrant's consumers (see Eq.(4)). The magnitude of this negative direct effect increases with the sensitivity of seller demand.

Second, a higher access charge increases the entrant's price, which reduces the entrant's demand. This indirect effect has an ambiguous impact on the platform's incentives to increase its price for sellers and its magnitude increases the sensitivity of the entrant's demand. Since $\partial^2 \pi^{PF} / \partial p_S \partial p_E = (\partial D_B^E / \partial p_E)(D_S(p_S) + (\partial D_S / \partial p_S)(a + p_S - c))$, a higher entrant's price provides the platform with an incentive to decrease its price for sellers if the sensitivity of seller demand is low. Therefore, if the sensitivity of seller demand is low, the indirect effect is negative. If the sensitivity of seller demand is high, the platform's incentive to decrease its price for sellers can either increase or decrease when the entrant's price increases, depending on the level of the access charge.

Therefore, if the sensitivity of the entrant's demand is low, the indirect effect is of low magnitude, and the seller price decreases with the access charge. If the sensitivity of the entrant's demand is high and if the sensitivity of seller demand is low, the direct and the indirect effect are negative, and therefore, the seller price decreases with the access charge. Lastly, if the sensitivity of seller demand is high and if the sensitivity of the entrant's demand is high, the platform's price for sellers depends on a trade-off between a negative effect and a positive effect: the platform's price for sellers first may first decrease and then increase with the access charge.

An example with linear demands: Consider the previous example of linear demands. If $\gamma_S = \gamma_B = \delta_E = 0.8$, the platform's price for sellers decreases with the access charge for low values

of it and then increases with the access charge.

4.1.1 Stage 2: the entrant's decision to enter the market

At stage 2, the entrant decides to enter the buyer market if it makes positive profits; that is, if

$$D_S(p_S^e(a))D_B^E(p_E^e(a))(p_E^e(a) - a - c_E) - \phi \geq 0.$$

The entrant's decision to enter the buyer market depends on the access charge, which increases its marginal cost, and on the price that the platform charges to sellers, which impacts its transaction volume. Since, from Lemma 2, the entrant's price decreases with a , there exists a maximum level of the access charge, denoted by \tilde{a} , such that $D_B^E(p_E^e(\tilde{a})) = 0$. Therefore, if there is entry, the access charge a is necessarily lower than \tilde{a} .

The impact of the access charge on the entrant's profits: To study the impact of the access charge on the entrant's profits, I totally differentiate the entrant's profit with respect to a . From the envelope theorem, I have

$$\frac{d\pi^E}{da} = \left. \frac{\partial \pi^E}{\partial a} \right|_{P^*} + \left. \frac{\partial \pi^E}{\partial p_S} \right|_{P^*} \frac{dp_S^e}{da} \Big|_{P^*}.$$

The access charge affects the entrant's profit through a direct and an indirect effect. Table 1 below summarizes the direct and the indirect effects in the case of linear demands.

Table1:summary of the direct and the indirect impact of the access charge on the entrant's profits with linear demands.

	Low sensitivity of merchant demand	High sensitivity of merchant demand
Low sensitivity of entrant's demand	Direct effect (-) Indirect effect (+) but of low magnitude	Direct effect (-) Indirect effect (-) for high values of F
High sensitivity of entrant's demand	Direct effect (-) Indirect effect (+) but of low magnitude	Direct effect (-) Indirect effect (+)

First, the access charge has a negative direct effect on the entrant's profit. A higher access charge increases the entrant's marginal cost, which reduces its profits. Second, the access charge has an indirect effect on the entrant's profits, because it affects the price that the platform chooses

for sellers at stage 3. Since $\partial\pi^E/\partial p_S = (p_E - a - c_E)(\partial D_S/\partial p_S)D_B^E < 0$, the sign of the indirect effect depends on the impact of the access charge on the seller price. As shown in Lemma 2, if the sensitivity of seller demand and the sensitivity of the entrant's demand are high, the seller price increases with the access charge for high values of it. In all other cases, the entrant's price decreases with the access charge. Therefore, if the sensitivity of seller demand and the sensitivity of the entrant's demand are high, the indirect effect is negative, and the entrant's profits decrease with the access charge for high values of it. Otherwise, the indirect effect is positive. The variations of the entrant's profits with the access charge depend on how the direct effect and the indirect effect compensate each other. The magnitude of the indirect effect depends on the sensitivity of seller demand, because $\partial D_S/\partial p_S$ appears in $\partial\pi^E/\partial p_S$. Therefore, if the sensitivity of seller demand is low, the direct effect is dominant, and the entrant's profits decrease with the access charge. If the sensitivity of seller demand is high and if the sensitivity of the entrant's demand is low, the entrant's profits increase with the access charge for low values of it and then increase with the access charge. Indeed, the indirect effect can dominate the direct effect for low values of the access charge because the entrant's margin is high. When the access charge increases, the indirect effect becomes lower, and the entrant's profits decrease with the access charge.

I denote the access charge that maximizes the entrant's profit by $\hat{a} \in [0, \tilde{a}]$.

An example with linear demands: In our previous example, if $\gamma_S = \gamma_B = \delta_E = 1$, the entrant's profits at the equilibrium of stage 3 are

$$\pi^E(a) - \phi = \frac{1}{12} (1 - a)^2 \left(6 - 2a - \sqrt{2}\sqrt{9 - 12a + 5a^2} \right) - \phi.$$

In this case, the entrant's profits are convex and decreasing with a for $a \in [0, 1]$. However, we can find examples in which the entrant's profits are increasing with the access charge if the sensitivity of seller demand is high. For example, if $\gamma_S = \gamma_B = 0.8$ and $\delta_E = 0.1$, we find that the entrant's profits increase with a for low values of the access charge and then decrease with a . The seller price is decreasing with a . Therefore, if the entrant's demand is not very sensitive to prices, and if the seller and buyer demand are relatively more sensitive to prices, we can find cases in which the entrant increases its profits by paying a high access charge to the platform. Indeed, a higher access charge lowers the seller price, and, therefore, increases its transaction volume.

4.1.2 Stage 1: the platform's decision to open its infrastructure

I am now able to study the platform's decision to open its infrastructure at stage 1. The platform compares its profits if the entrant enters and if it does not enter. I denote by $a^* \in [0, \tilde{a}]$ the access charge that maximizes the platform's profit under the constraint that the entrant enters the market. There are three cases:

1. Blockaded entry: the entrant never enters the market, because for all $a \in [0, \tilde{a}]$, we have

$$\pi^E(p_S^e(a), p_E^e(a), a) - \phi < 0.$$

2. Entry accommodation: the platform chooses the access charge that maximizes its profits under the constraint that the entrant enters the market; that is, such that

$$\pi^E(p_S^e(a^*), p_E^e(a^*), a^*) - \phi \geq 0.$$

3. Entry deterrence: the platform chooses a level of the access charge a^d that deters entry; that is, such that

$$\pi^E(p_S^e(a^d), p_E^e(a^d), a^d) - \phi < 0.$$

I start by analyzing the impact of the access charge on the platform's profits in case of entry. I totally differentiate the platform's profit in case of entry with respect to a . From the envelope theorem, we have that

$$\left. \frac{d\pi^{PF}}{da} \right|_{P^*} = \left. \frac{\partial \pi^{PF}}{\partial a} \right|_{P^*} + \left. \frac{\partial \pi^{PF}}{\partial p_E} \right|_{P^*} \left. \frac{\partial p_E}{\partial a} \right|_{P^*}.$$

There are two channels through which the access charge impacts the platform's profits. First, since $\partial \pi^{PF} / \partial a = D_B^E(p_E) \geq 0$, the access charge has a direct impact on the platform's profits. Indeed, a higher access charge increases the platform's benefits per consumer on the entrant's market. Second, the access charge has an indirect impact on the platform's profits because it affects the price that the entrant chooses at stage 3 and, therefore, the revenues that the platform earns from the entrant's consumers. The indirect effect is negative if the platform earns positive revenues from the entrant's consumers (that is, if $a + p_S^e - c_S > 0$), because $\partial p_E / \partial a > 0$ and $\partial \pi^{PF} / \partial p_E = (\partial D_B^E / \partial p_E)(a + p_S^e - c_S) < 0$. Otherwise, if the platform incurs losses on the entrant's transactions, the indirect effect is positive, and the platform's profits increase with the access

charge. The magnitude of the indirect effect depends on the sensitivity of consumer demand on the entrant's market (because $\partial D_B^E / \partial p_E$ appears in $\partial \pi^{PF} / \partial p_E$). Therefore, if the sensitivity of consumer demand on the new market is low, the platform's profits increase with the access charge, because the magnitude of indirect effect is low. By contrast, if the sensitivity of consumer demand on the new market is high, the indirect effect may become dominant, and the platform's profits may decrease with the access charge (see the example below).

If entry is not blockaded, the platform chooses between the strategies "entry deterrence" and "entry accommodation". The platform chooses to accommodate entry if

$$\pi^{PF}(p_B^e(a^*), p_S^e(a^*), p_E^e(a^*), a^*) > \pi^{PF}(p_B^m, p_S^m). \quad (8)$$

Note that, if the entrant's profits decrease with the access charge, the platform must increase the access charge to deter entry. However, the entrant's profits may also increase with the access charge in my setting because the entrant values the presence of sellers on the platform. Therefore, in some cases, entry could be deterred by lowering the access charge.

In Proposition 3, I provide sufficient conditions for the platform to accommodate entry. I denote by \bar{a} the highest level of the access charge such that the entrant enters the market.

Proposition 3 *If the platform incurs losses on the entrant's transactions (that is, if for all $a \in [0, \bar{a}]$, we have $a + p_S^e(a) - c_S < 0$) and if $\pi^{PF}(p_B^e(\bar{a}), p_S^e(\bar{a}), p_E^e(\bar{a}), \bar{a}) > \pi^{PF}(p_B^m, p_S^m)$, the platform accommodates entry and the entrant makes zero profits. If there exists $a \in (0, \bar{a})$ such that $p_B^m \leq a \leq \bar{a}$ and $a + p_S^e(a) - c_S > 0$, the platform accommodates entry.*

In Proposition 3, I demonstrate that if sellers are heavily subsidized (that is, if the price for sellers is lower than the platform's net cost of serving the entrant's consumers), the platform accommodates entry if its profits with the highest access charge that triggers entry are higher than in the monopoly case. If there exists a level of the access charge such that the platform makes more profits on the new market than on the initial market, the platform always prefers to accommodate entry.

An example with linear demands: In our example, if $\gamma_S = \gamma_B = \delta_E = 1$, if $a \in [0, 1]$, the platform's profits at the equilibrium of stage 2 are

$$\pi^{PF} = \frac{1}{54} \left(6 - 2a - \sqrt{2}\sqrt{\psi} \right) \left(3\sqrt{2}\sqrt{\psi} + a(12 - 8a - \sqrt{2}\sqrt{\psi}) \right),$$

where $\psi = 9 - 12a + 5a^2 > 0$. The platform's profits are concave in a for $a \in [0, 1]$ and reach a maximum at $a^* = 1/3$. If we assume that the entrant's entry cost is arbitrarily small, if the platform wants to deter entry, it must choose $a^d = 1$. The platform's profits if it chooses to deter entry are equal to $8/27$ (the monopoly profit) and the platform's profits if it chooses to accommodate entry are equal to $4/9$. Therefore, the platform chooses to accommodate entry in this example.

If $\gamma_S = \gamma_B = 1$ and $\delta_E = 0.1$, the platform's profits increase with the access charge. The entrant's profits increase and then decrease with the access charge. The platform's optimal strategy is to choose the highest access charge that triggers entry. If the entrant's fixed cost of entering the market is not too high, the entrant enters and makes zero profit. Note that even if the platform chooses an access charge that is equal to zero, the platform's profits are higher if there is entry than in the monopoly case ($0.41 > 8/27$), because the platform earns higher revenues from sellers ($p_S(0) = 0.41 > p_S^m = 1/3$ and $p_B(0) = 0.29 < p_B^m = 1/3$). Therefore, if the sensitivity of the entrant's demand is low, the platform chooses to accommodate entry on the buyer side of the market because it can extract more rents from the seller side.

If $\gamma_S = \gamma_B = 0.1$ and $\delta_E = 0.8$, the platform's profits and the entrant's profits decrease with the access charge. The platform's optimal strategy is to choose $a = 0$ and the entrant enters the market if the entry cost is not too high.

Finally, remark that it could even be profitable for the platform to subsidize access by choosing a negative access charge, in order to extract more rents from sellers.

4.1.3 The socially optimal access charge

In this subsection, I assume that a regulator sets the access charge that maximizes the sum of consumer surplus, seller surplus, platform's profits, and entrant's profits. I determine the welfare-maximizing charge in my example with linear demands.

The net surplus on each side of the market for an average transaction is

$$V_k(p_k) = \int_{p_k}^{+\infty} D_k(t) dt,$$

for $k = S, B$. The welfare-maximizing access charge maximizes social welfare under the constraint that the platform makes a positive profit. The social welfare is

$$W = V_S(p_S) [D_B^{PF}(p_B) + D_B^E(p_E)] + V_B^{PF}(p_B) D_S(p_S) + V_B^E(p_E) D_S(p_S) + \pi^{PF}.$$

If $\gamma_S = \gamma_B = 0.1$ and $\delta_E = 0.8$, social welfare is decreasing with the access charge if there is entry. The platform's profits and the entrant's profits are decreasing with the access charge, the buyer prices increase with the access charge, whereas the seller price varies non-monotonically with the access charge. The potential positive impact of the access charge on seller demand does not compensate the fall in firms' profits and consumer surplus. Consequently, it is socially optimal to set the access charge to zero in case of entry. The profit-maximizing access charge is also socially optimal.

If $\gamma_S = \gamma_B = 0.8$ and $\delta_E = 0.1$, social welfare is increasing with the access charge if there is entry. The platform's profits increase with the access charge, the entrant's profits vary non-monotonically with the access charge. The buyer prices increase with the access charge, but the seller price decreases with the access charge. Consequently, it is socially optimal to set the access fee such that the entrant makes zero profit in case of entry. The profit-maximizing access charge is also socially optimal.

If $\gamma_S = \gamma_B = 0.201$ and $\delta_E = 0.1$, social welfare is first increasing and then decreasing with the access charge. The platform's profits increase with the access charge, whereas the entrant's profits decrease with the access charge. Since the profit-maximizing access charge is equal to \bar{a} , the profit-maximizing access charge is too high to maximize social welfare.

5 General case: competition for buyers

In this section, I analyze the general case in which the entrant competes with the platform by offering differentiated access services.

5.1 Stage 3: prices

At stage 3, if there is entry, the platform and the entrant compete in prices. The platform and the entrant choose the prices that maximize their profits given by (1) and (2), respectively. Solving for the first-order conditions of platform's profit-maximization yields

$$D_S(p_S) \left(\frac{dD_B^{PF}}{dp_B}(p_B + p_S - c) + D_B^{PF}(p_B, p_E) + \frac{dD_B^E}{dp_B}(a + p_S - c_S) \right) = 0, \quad (9)$$

and

$$D_S(p_S) D_B(p_B, p_E) + \frac{dD_S}{dp_S} (D_B^{PF}(p_B, p_E)(p_B + p_S - c) + D_B^E(p_B, p_E)(a + p_S - c_S)) = 0. \quad (10)$$

Solving for the first-order condition of entrant's profit maximization yields

$$D_S(p_S) \left(D_B^E(p_B, p_E) + \frac{dD_B^E}{dp_E}(p_E - a - c_E) \right) = 0. \quad (11)$$

I denote by $p_B^{BR}(p_E)$ and $p_S^{BR}(p_E)$ the best-responses functions of the platform to the price chosen by the entrant, and by $p_E^{BR}(p_B)$ is the best-response function of the entrant to the price chosen by the platform on the buyer side. There are two main differences with respect to the benchmark. First, the platform's price for buyers depends on the price chosen by the entrant (not only through the seller price). Second, the entrant's best-response depends on the price chosen by the platform for buyers, but not for sellers. Therefore, there are three kinds of externalities in the general case:

- (i) The externality that the platform exerts on the entrant through the choice of the price for sellers: as in the benchmark, a lower seller price increases the transaction volume.
- (ii) The externality that the entrant exerts on the platform's interconnection revenues through the choice of its price: as in the benchmark, if the entrant reduces its price, it increases the profits that the platform earns on the entrant's transactions.
- (iii) The competitive externality that the platform and the entrant exert on each other when they choose their prices for buyers.

The best-response functions are implicitly defined by

$$\frac{\partial \pi^{PF}}{\partial p_B}(p_B^{BR}, p_S^{BR}) = \frac{\partial \pi^{PF}}{\partial p_S}(p_B^{BR}, p_S^{BR}) = \frac{\partial \pi^E}{\partial p_E}(p_E^{BR}) = 0.$$

In the Appendix, I show that the best-response of the entrant is always increasing with the platform's price for buyers and I provide a condition under which prices are strategic complements on the buyer side. This condition is sufficient to ensure that there exists a Nash equilibrium at stage 3.⁸

I denote by $P^e = (p_B^e, p_S^e, p_E^e)$ the prices chosen at the Nash equilibrium of the price setting subgame.

⁸The platform's best-response functions are not necessarily uniquely defined by the first-order conditions. However, in the linear demand case, it is possible to select the prices that maximize the platform's profit by using the second-order conditions or by evaluating the platform's profit at the candidate solutions.

An example: To illustrate my model setting, following Dixit (1979) and Singh and Vives (1984), I use the following demand functions, which satisfy assumption 1:

$$D_B^{PF}(p_B, p_E) = 1 - p_B + (1 - b)p_E,$$

and

$$D_B^E(p_B, p_E) = 1 - p_E + (1 - b)p_B.$$

These demand functions correspond to a Bertrand duopoly setting with differentiated products and the parameter $b \in [0, 1]$ represents the degree of product differentiation.⁹ I also assume that $D_S(p_S) = 1 - \gamma_S p_S$, where $\gamma_S > 0$ is a parameter that captures the sensitivity of merchant demand. It is possible to compute the Nash equilibrium of the subgame with these demand functions. However, due to the algebraic complexity of the results, I only use them to compare the equilibrium prices and to study the variation of firms' profits with respect to the access charge.

It is interesting to analyze the impact of entry of the price structure and on the difference between the platform's price for buyers and the entrant's price. In the competition case, the platform's incentives to increase its price for buyers increase with the degree of product differentiation. Compared to the benchmark, Eq.(9) contains an additional term, which reflects the impact of the buyer price on the profits earned from the entrant's consumers. If the degree of product differentiation is low, the platform's profits earned from the entrant's consumers are very sensitive to the platform's price for buyers, which increases the entrant's demand. Therefore, if the degree of product differentiation is low, the platform's incentives to increase its price for buyers are higher than in the benchmark case, because the platform obtains higher revenues from the entrant's consumers. Proposition 4 compares the platform's price for buyers and the platform's price for sellers in our example if there is entry. It also compares the platform's price for buyers to the entrant's price.

Proposition 4 *In a Bertrand duopoly setting with differentiated products on the buyer side, if the degree of product differentiation is low and if the sensitivity of seller demand is high, the platform charges a higher price to buyers than to sellers if the entrant enters the market. If the degree of product differentiation and the sensitivity of seller demand are high, the platform charges a higher price to sellers if the access charge is low, and a higher price to buyers if the access charge is high. If*

⁹These demand functions are obtained by assuming that a representative consumer maximizes the utility $U(q_1, q_2) = q_1 + q_2 - (1/2)(q_1^2 + 2(1 - b)q_1q_2 + q_2^2)$, where $1 - (1 - b)^2 > 0$ to ensure demand concavity. In this case, consumers have a taste for variety. The system of demand functions is obtained by maximizing consumer utility and by inverting the demand system.

the sensitivity of seller demand is low, the platform charges a lower price to buyers than to sellers. The price charged by the platform to buyers is always lower than the entrant's price.

Proposition 4 shows that the degree of product differentiation impacts the comparison of the buyer price and the seller price. If the degree of product differentiation is high, we have the same result as in the benchmark; that is, the platform's price structure is biased in favor of sellers for high values of the access charge. If the degree of product differentiation is low, the price charged to buyers is always higher than the price charged to sellers, because the platform obtains revenues from the entrant's consumers. One policy implication of this result is that entry in two-sided markets can stimulate competition on one side of the market, while reinforcing the market power of the platform on the other side.

Comparative statics: I am now able to analyze the impact of the access charge on the equilibrium prices that are chosen at stage 3.

Lemma 4 *If there is entry, the entrant's price and the platform's price for buyers increase with the access charge. The platform's price for sellers may either increase or decrease with the access charge.*

As in the benchmark, the entrant's price increases with the access charge. However, it now depends on three effects. First, as in our benchmark, when the access charge increases, the entrant's marginal cost increases and, therefore, the entrant has an incentive to increase its price. Second, a higher access charge impacts the buyer price. The platform has an incentive to increase its price for buyers when the access charge increases, because it obtains higher revenues from the new market. Since prices are strategic complements on the buyer market, if the platform's price for buyers increases, the entrant has an incentive to increase its price. Third, a higher access charge impacts the seller price. The platform has incentives to lower its price for sellers, to increase the revenues obtained on the new market if the access charge is high. Since a higher price for sellers implies a bias of the price structure, the platform increases its price for buyers. Since prices are strategic complements on the buyer market, this provides the entrant with another incentive to increase its price. A similar analysis shows that the platform's price for buyers increases with the access charge. Therefore, the impact of the access charge is to soften the competition that takes place between the platform and the entrant at stage 3. The analysis of the impact of the access charge on the seller price is similar to the benchmark.

5.2 Stage 2: entry

As in the benchmark, at stage 2, the entrant decides to enter the market for buyers if it makes positive profits; that is, if

$$D_S(p_S^e(a))D_B^E(p_B^e(a), p_E^e(a))(p_E^e(a) - a - c_E) - \phi \geq 0.$$

The entrant's decision to enter the market for buyers depends on the access charge, which increases its marginal cost, and on the price that the platform charges to sellers, which impacts its transaction volume, as in the benchmark. Indeed, a higher seller price lowers the entrant's incentives to enter the market. Furthermore, in the competition case, the entrant's decision to enter also depends on the platform's price for buyers, which impacts its demand.

I totally differentiate the entrant's profit with respect to a . From the envelope theorem, we have

$$\frac{d\pi^E}{da} = \frac{\partial \pi^E}{\partial a} \Big|_{P^*} + \frac{\partial \pi^E}{\partial p_S} \Big|_{P^*} \frac{dp_S^e}{da} \Big|_{P^*} + \frac{\partial \pi^E}{\partial p_B} \Big|_{P^*} \frac{dp_B^e}{da} \Big|_{P^*}.$$

There are two channels through which the access charge can affect the entrant's profits. First, the access charge impacts the entrant's profits through a direct effect, which is negative as in the benchmark. Second, the access charge has an indirect impact on the entrant's profits, because it affects the prices that the platform chooses at stage 3. The analysis of the impact of the seller price on the entrant's profits is similar to the benchmark. In the competition case, unlike in the benchmark, the platform's price for buyers impacts the entrant's profits. Since $dp_B^e/da > 0$ and $\partial \pi^E / \partial p_B = D_S(p_S)(\partial D_B^E / \partial p_B)(p_E - a - c_E) > 0$, the indirect effect of the access charge on the entrant's profits (through the platform's choice of the buyer price) is positive. Indeed, from Lemma 2, a higher access charge increases the platform's price for buyers, which increases the entrant's profits. The magnitude of this effect depends on the sensitivity of the entrant's demand to the platform's price for buyers (see the example below).

An example: In my example of a Bertrand duopoly setting with differentiated products, if the degree of product differentiation is low (e.g., $b = 0.1$) and if the sensitivity of seller demand is high (e.g., $\gamma_S = 1$), the entrant's profits increase with the access charge for low values of the access charge and then decrease with the access charge. In this case, the entrant's price is very sensitive to the platform's price for buyers (as $\partial D_B^E / \partial p_B = 1 - b > 0$) and, therefore, the indirect impact of the

access charge on the entrant's profits through the platform's choice of the buyer price is dominant for low values of the access charge. If the degree of product differentiation is high (e.g., for $b = 0.5$ or $b = 0.9$), the entrant's profits decrease with the access charge.

5.3 Stage 1: the platform's decision to open its infrastructure

At stage 1, the platform decides whether to open its infrastructure to the entrant. If it opens its infrastructure, the platform chooses the level of the access charge a that maximizes its profit under the constraint that the entrant enters the market. The choice of the access charge depends on its direct effect on the platform's profit, and on its strategic impact on the competition that takes place at stage 3. From the envelope theorem, we have

$$\frac{d\pi^{PF}}{da} = \frac{\partial\pi^{PF}}{\partial a} + \frac{\partial\pi^{PF}}{\partial p_E} \Big|_{P^e} \frac{\partial p_E^e}{\partial a}. \quad (12)$$

Since $\partial\pi^{PF}/\partial a = D_B^E(p_B, p_E)D_S(p_S) > 0$, the direct effect is positive. A higher access charge increases the platform's interconnection revenues. The indirect effect depends on the impact of the access charge on the price that is chosen by the entrant at the equilibrium of stage 3 and on the impact of the entrant's price on the platform's profit. We have

$$\frac{\partial\pi^{PF}}{\partial p_E} \Big|_{P^e} = D_S(p_S^e) \left(\underbrace{\frac{\partial D_B^{PF}}{\partial p_E} \Big|_{P^e} (p_B^e + p_S^e - c)}_{\text{Term I}} + \underbrace{\frac{\partial D_B^E}{\partial p_E} \Big|_{P^e} (a + p_S^e - c_S)}_{\text{Term II}} \right). \quad (13)$$

Since $\partial D_B^{PF}/\partial p_E > 0$ and $\partial D_B^E/\partial p_E < 0$, the entrant's price has an ambiguous impact on the platform's profits, which depends on the relative revenues generated by the platform's consumers (Term I of (13)) and the revenues generated by the entrant's consumers (Term II of (13)). In the benchmark, the demand of the platform's consumers does not depend on the entrant's price, and the first term of (13) is equal to zero. A higher rival price increases the platform's revenues from its consumers, as their demand increases, whereas it lowers the revenues from the entrant's consumers, provided that the margin per new consumer is positive (that is, $a + p_S^e - c_S > 0$). The magnitude of the indirect effect depends on the sensitivity of consumer demand to the entrant's price and the sensitivity of the entrant's demand (see the example below).

If the platform's margin on the entrant's consumers is negative, the indirect effect of the access charge on the platform's profit is positive. Therefore, the platform's profits increase with the access

charge, and, as in the benchmark, if the platform accommodates entry, the entrant makes zero profit. The platform's margin on the entrant's consumers depends on whether the seller side of the market is subsidized at the equilibrium.

An example: In my example of a Bertrand duopoly setting with differentiated products, I find that, if $b = 0.1$, the platform's profits increase with the access charge. The platform's optimal strategy is to choose the maximum level of the access charge that triggers entry on the market, because it makes more profit than in the monopoly case. This finding is consistent with my analysis of the impact of the access charge on the platform's profits. If the degree of product differentiation is low ($b = 0.1$), the indirect effect is of low magnitude if it is negative, or it can even be positive, because the platform earns more profits when the entrant's price increases. Hence, the platform's profits increase with the access charge. If the degree of product differentiation is high ($b = 0.9$), the platform's profits increase with the access charge until $a = 0.4$ and then decrease with the access charge. In this case ($b = 0.9$), the indirect effect is negative, because the platform's demand is not very sensitive to the variations of the entrant's price. Therefore, the platform's incurs losses when the entrant's price increases, because its profits on the entrant's consumers decrease, whereas its profits on its own consumers do not increase because the products offered by the platform and the entrant are differentiated. For low values of the access charge, the losses on the entrant's consumers are low. Therefore, the direct effect dominates the indirect effect and the platform's profits increase with the access charge. When the access charge is higher than 0.4, the indirect effect is dominant, and the platform's profits decrease with the access charge.

The impact of the seller fee on the platform's interconnection revenues is specific to the case of two-sided markets, and is absent from the literature on telecommunications networks. Another way of analyzing Eq.(13) is to account for the impact of the entrant's price on the platform's profit as the sum of its effects on the revenues from the buyer side and on the seller side, that is

$$\left. \frac{\partial \pi^{PF}}{\partial p_E} \right|_{P^e} = D_S(p_S^e) \left(\left. \frac{\partial D_B^{PF}}{\partial p_E} \right|_{P^e} p_B^e + \left. \frac{\partial D_B^E}{\partial p_E} \right|_{P^e} a + \left. \frac{\partial D_B}{\partial p_E} \right|_{P^e} (p_S^e - c_S) \right) \quad (14)$$

Eq.(14) shows that an important condition for the seller fee to impact the choice of the access charge is that the total market size on the buyer side ($D_B = D_B^{PF} + D_B^E$) is not fixed. Therefore, it seems interesting to analyze the special case in which entry does not generate any expansion of the buyer market.

A special case: no market expansion effect on the buyer side: If the total size of the buyer market is fixed, entry does not generate additional traffic on the platform. The effect of competition is only to move market shares from the platform to the entrant. This implies that

$$\frac{d\pi^{PF}}{da} = D_S(p_S^e) \left(D_B^E(p_B^e, p_E^e) + \frac{\partial D_B^{PF}}{\partial p_E} \Big|_{P^e} (p_B^e - a) \frac{\partial p_E^e}{\partial a} \right).$$

From (11), the equation above can be rearranged as

$$\frac{d\pi^{PF}}{da} = D_S(p_S^e) \frac{\partial D_B^{PF}}{\partial p_E} \Big|_{P^e} \left(p_E^e - a - c_E + (p_B^e - a) \frac{\partial p_E^e}{\partial a} \right).$$

Therefore, if the access charge is lower than the platform's price for buyers, the platform's profits increase with the access charge if there is entry (that is, if $a < p_E^e + c_E$).

If the access charge is higher than the platform's price for buyers, I focus on the special case in which $p_B^e = p_E^e$. The platform's profits increase with the access charge for $a < p_B^e$ and decrease with the access charge for $a > p_B^e$. Consequently, the platform's profits reach a maximum at $a = p_B^e$. This implies that, if $p_B^e = p_E^e$, the entrant does not enter if the platform chooses the access charge that maximizes its profits. Therefore, if the platform accommodates entry, the platform sets $a = \bar{a}$, where \bar{a} denotes the maximum level of the access charge that triggers entry on the market.

(To revise)

Proposition 5 *If the total size of the market is fixed, and if the prices chosen by the entrant and by the platform for buyers are identical at the equilibrium of stage 3, the platform accommodates entry if $\pi^{PF}(p_B^e(\bar{a}), p_S^e(\bar{a}), p_E^e(\bar{a}), \bar{a}) \geq \pi^{PF}(p_B^m, p_S^m)$ and the entrant makes zero profit. If $\pi^{PF}(p_B^e(\bar{a}), p_S^e(\bar{a}), p_E^e(\bar{a}), \bar{a}) < \pi^{PF}(p_B^m, p_S^m)$, the platform makes more profits if it deters entry.*

An important consequence of Proposition 5 is that, if the platform and the entrant's price are identical, the platform has no incentives to open its market to the entrant if the market expansion effect is not sufficient to compensate its loss of sales, unless it can charge sellers with a higher price than in the monopoly case.

6 Perfect competition on the new market

In this section, I assume that the entrants are perfectly competitive and that they operate on a separate market. There are no fixed entry costs ($\phi = 0$). Therefore, in this case, the price that is

charged by the entrants at the equilibrium of the price setting subgame is $p_E^e = a + c_E$.

The profit-maximizing access charge I start by studying the profit-maximizing access charge. From Eq. (12) and (13), since $\partial p_E^e / \partial a = 1$ and $\partial \pi^{PF} / \partial a = D_B^E(p^E)$, the profit maximizing access charge solves

$$D_B^E(p_E) + (a + p_S^e - c) \frac{\partial D_B^E}{\partial p_E} = 0.$$

Therefore, we have

$$a^* = c - p_S^e + \frac{p_E}{\eta_E},$$

where $\eta_E = -(\partial D_B^E / \partial p_E)(p_E / D_B^E)$. In other words, the profit-maximizing access charge is equal to the sum of the platform's net cost of serving the entrant's consumers and a mark up, which reflects the revenues that the incumbent platform can obtain from the access activities. If the demand elasticity on the new market is high, then the profit-maximizing access charge is close to the platform's net cost of serving the entrant's consumers (that is, the platform's marginal cost, minus the price that the platform obtains from sellers). The logic of this result is similar to the literature on one-way access in telecommunications networks (see Laffont and Tirole, 2001). The profit-maximizing pricing of access should reflect conditions of demand in the retail market. Since the entrants are perfectly competitive, the platform can use the access charge to price discriminate between the two market segments on the buyer side. However, in my setting, the net cost of serving the entrant's consumers depends on the price that the platform sets for sellers.

Since $(p_B + p_S - c) / p_B = 1 / \eta_B^{PF}$, the choice of the profit-maximizing access charge implies that the entrant's price is

$$p_E = \frac{c_E + (1 - \frac{1}{\eta_B^{PF}}) p_B}{(1 - \frac{1}{\eta_E})}.$$

The welfare-maximizing prices: If the constraint that $\pi^{PF} \geq 0$ is not binding, the welfare-maximizing prices are such that

$$p_B - c_B = p_E - c_E,$$

and

$$\frac{p_S}{p_B} = \frac{V_B^{PF}}{V_S} \frac{D_S}{D_B^{PF} + D_B^E}.$$

As in Rochet and Tirole (2003), the welfare-maximizing price structure depends on the relative average surplus on each side of the market.

7 Conclusion

This paper is a first step towards understanding a platform's incentives to open its infrastructure on one side of the market to an entrant. Since the entrant values the presence of sellers on the platform, the entrant's profits may increase with the access charge for low values of it. If entry occurs, the platform's price structure is biased in favor of sellers if the degree of product differentiation is low. By contrast, if the degree of product differentiation is high, the platform distorts the price structure in favor of buyers for low values of the access charge. The platform may choose to accommodate entry by setting an access fee that is equal to zero if it can extract rents from sellers. The policy implication of this analysis is that, in two-sided markets, entry on one side may have negative welfare effects on the other side, because the platform may decide to accommodate entry on one side of the market to extract rents from the other side.

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Appendix

Proof of Lemma 1: Differentiating (3) and (4) with respect to p_E , I obtain that

$$\begin{pmatrix} \partial^2 \pi^{PF} / \partial p_B \partial p_E \\ \partial^2 \pi^{PF} / \partial p_E \partial p_S \end{pmatrix} = \begin{pmatrix} \partial^2 \pi^{PF} / \partial p_B^2 & \partial^2 \pi^{PF} / \partial p_B \partial p_S \\ \partial^2 \pi^{PF} / \partial p_B \partial p_S & \partial^2 \pi^{PF} / \partial p_S^2 \end{pmatrix} \begin{pmatrix} \partial p_B^{BR} / \partial p_E \\ \partial p_S^{BR} / \partial p_E \end{pmatrix}.$$

By inverting this system of equations, we have

$$\begin{pmatrix} \partial p_B^{BR} / \partial p_E \\ \partial p_S^{BR} / \partial p_E \end{pmatrix} = -\frac{1}{\Delta_0} \begin{pmatrix} \partial^2 \pi^{PF} / \partial p_S^2 & -\partial^2 \pi^{PF} / \partial p_B \partial p_S \\ -\partial^2 \pi^{PF} / \partial p_B \partial p_S & \partial^2 \pi^{PF} / \partial p_B^2 \end{pmatrix} \begin{pmatrix} \partial^2 \pi^{PF} / \partial p_B \partial p_E \\ \partial^2 \pi^{PF} / \partial p_E \partial p_S \end{pmatrix},$$

where $\Delta_0 = (\partial^2 \pi^{PF} / \partial p_B^2)(\partial^2 \pi^{PF} / \partial p_S^2) - (\partial^2 \pi^{PF} / \partial p_S \partial p_B)^2 > 0$ from Assumption 2. Since $\partial^2 \pi^{PF} / \partial p_B \partial p_E = 0$, this implies that

$$\frac{\partial p_B^{BR}}{\partial p_E} = \frac{1}{\Delta_0} \left(\frac{\partial^2 \pi^{PF}}{\partial p_B \partial p_S} \frac{\partial^2 \pi^{PF}}{\partial p_E \partial p_S} \right),$$

and

$$\frac{\partial p_S^{BR}}{\partial p_E} = -\frac{1}{\Delta_0} \left(\frac{\partial^2 \pi^{PF}}{\partial p_B^2} \frac{\partial^2 \pi^{PF}}{\partial p_S \partial p_E} \right).$$

Since the platform's profit is concave in its prices and since $\Delta_0 > 0$, we have $\partial^2 \pi^{PF} / \partial p_B^2 < 0$ and $\partial^2 \pi^{PF} / \partial p_B \partial p_S < 0$. Therefore, $\partial p_B^{BR} / \partial p_E$ has the sign of $-\partial^2 \pi^{PF} / \partial p_E \partial p_S$, and $\partial p_S^{BR} / \partial p_E$ has the sign of $\partial^2 \pi^{PF} / \partial p_E \partial p_S$. This implies that the entrant's price impacts the buyer price and the seller price in opposite directions. By differentiating (4) with respect to p_E , we have

$$\frac{\partial^2 \pi^{PF}}{\partial p_S \partial p_E} = \frac{\partial D_B^E}{\partial p_E} \left(D_S + (F + p_S - c) \frac{\partial D_S}{\partial p_S} \right).$$

Since $\partial D_B^E / \partial p_E < 0$ and $\partial D_S / \partial p_S < 0$, if $a < c - p_S^{BR}$, we have $\partial^2 \pi^{PF} / \partial p_S \partial p_E < 0$. This implies that, if $a < c - p_S^{BR}$, we have $\partial p_B^{BR} / \partial p_E > 0$ and $\partial p_S^{BR} / \partial p_S < 0$. This completes the proof of Lemma 1.

Proof of Proposition 1: If the platform is a monopoly on the buyer market, we have $D_B^E(p_E) = 0$. From (1), the first-order conditions of profit-maximization are

$$\frac{dD_B^{PF}}{dp_B}(p_B + p_S - c) + D_B^{PF} = 0, \quad (15)$$

and

$$D_S(p_S) + \frac{dD_S}{dp_S}(p_B + p_S - c) = 0. \quad (16)$$

The first-equation above enables me to define p_B as a function of the seller price p_S , that I denote by $\widetilde{p}_B(p_S)$. If the platform opens its infrastructure to an entrant, the buyer price is also defined by the same function of p_S (see (3)). However, the profit-maximizing p_S is different. Evaluating $\partial\pi^{PF}/\partial p_S$ given by (4) at $(p_S^m, \widetilde{p}_B(p_S^m))$, using the implicit definition of the monopoly seller price given by (16), we find that

$$\left. \frac{\partial\pi^{PF}}{\partial p_S} \right|_{(p_S^m, \widetilde{p}_B(p_S^m))} = D_B^E(p_E) \left[D_S(p_S^m) + \frac{dD_S}{dp_S}(p_S^m + \widetilde{p}_B(p_S^m) - c) \right]. \quad (17)$$

From (16), a monopolistic platform chooses p_S^m such that

$$D_S(p_S^m) = - \left. dD_S/dp_S \right|_{(p_S^m, \widetilde{p}_B(p_S^m))} (\widetilde{p}_B(p_S^m) + p_S^m - c).$$

Replacing for this expression in (17), we have

$$\left. \frac{\partial\pi^{PF}}{\partial p_S} \right|_{(p_S^m, \widetilde{p}_B(p_S^m))} = \left. \frac{dD_S}{dp_S} \right|_{(p_S^m, \widetilde{p}_B(p_S^m))} D_B^E(p_E) [a - \widetilde{p}_B(p_S^m)].$$

Since $\widetilde{p}_B(p_S^m) = p_B^m$, we have

$$\left. \frac{\partial\pi^{PF}}{\partial p_S} \right|_{(p_S^m, \widetilde{p}_B(p_S^m))} = \left. \frac{dD_S}{dp_S} \right|_{(p_S^m, p_B^m)} D_B^E(p_E) [a - p_B^m].$$

Let $g(p_S) \equiv \partial\pi^{PF}/\partial p_S|_{(p_S, \widetilde{p}_B(p_S))}$. It can be easily shown that g is decreasing in p_S . If $a - p_B^m > 0$ (resp. < 0), since $dD_S/dp_S < 0$, we have that $g(p_S^m) < 0 = g(p_S^{BR})$ (resp. > 0). Since g is decreasing in p_S , this implies that $p_S^m \geq p_S^{BR}(p_E)$ (resp. $p_S^m \leq p_S^{BR}(p_E)$).

Proof of Proposition 2: If $D_B^{PF} = 1 - \gamma_B p_B$, $D_S^{PF} = 1 - \gamma_S p_S$, $D_B^E = 1 - \delta_E p_E$, $c = c_E = 0$, and $\gamma_B = \gamma_S = \gamma$, we have

$$p_B^e - p_S^e = \frac{\gamma(4 - 2a\delta_E) - \sqrt{2\gamma^2(9 + a^2\delta_E(3\gamma + 2\delta_E) - 3a(\gamma + 3\delta_E))}}{2\gamma^2}.$$

Since $a < 1/\delta_E$, $p_B^e - p_S^e$ has the sign of

$$\begin{aligned} & \gamma^2(4 - 2a\delta_E)^2 + 2\gamma^2(9 + a^2\delta_E(3\gamma + 2\delta_E) - 3a(\gamma + 3\delta_E)) \\ &= 2\gamma^2(1 - a\delta_E)(-1 + 3a\gamma). \end{aligned}$$

If $a < 1/3\gamma$, we have $p_B^e - p_S^e < 0$. Otherwise, $p_B^e - p_S^e > 0$.

Proof of Lemma 2: By differentiating the first-order conditions with respect to a , we have

$$\begin{pmatrix} -\partial^2 \pi^{PF} / \partial p_B \partial a \\ -\partial^2 \pi^E / \partial p_E \partial a \\ -\partial^2 \pi^{PF} / \partial p_S \partial a \end{pmatrix} = \begin{pmatrix} \partial^2 \pi^{PF} / \partial p_B^2 & \partial^2 \pi^{PF} / \partial p_B \partial p_E & \partial^2 \pi^{PF} / \partial p_B \partial p_S \\ \partial^2 \pi^E / \partial p_B \partial p_E & \partial^2 \pi^E / \partial p_E^2 & \partial^2 \pi^E / \partial p_E \partial p_S \\ \partial^2 \pi^{PF} / \partial p_B \partial p_S & \partial^2 \pi^{PF} / \partial p_E \partial p_S & \partial^2 \pi^{PF} / \partial p_S^2 \end{pmatrix} \begin{pmatrix} dp_B^e / da \\ dp_E^e / da \\ dp_S^e / da \end{pmatrix}.$$

Denoting by M the matrix of cross-derivatives, we have

$$\begin{pmatrix} dp_B^e / da \\ dp_E^e / da \\ dp_S^e / da \end{pmatrix} = M^{-1} \begin{pmatrix} -\partial^2 \pi^{PF} / \partial p_B \partial a \\ -\partial^2 \pi^E / \partial p_E \partial a \\ -\partial^2 \pi^{PF} / \partial p_S \partial a \end{pmatrix}, \text{ where}$$

$$M^{-1} = \frac{1}{\det M} \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}.$$

Since $\partial^2 \pi^{PF} / \partial p_B \partial a = 0$, it follows that

$$\begin{aligned} dp_B^e / da &= \frac{-1}{\det M} (a_2 \partial^2 \pi^E / \partial p_E \partial a + a_3 \partial^2 \pi^{PF} / \partial p_S \partial a), \\ dp_E^e / da &= \frac{-1}{\det M} (b_2 \partial^2 \pi^E / \partial p_E \partial a + b_3 \partial^2 \pi^{PF} / \partial p_S \partial a), \\ dp_S^e / da &= \frac{-1}{\det M} (c_2 \partial^2 \pi^E / \partial p_E \partial a + c_3 \partial^2 \pi^{PF} / \partial p_S \partial a). \end{aligned}$$

Since $\partial^2 \pi^E / \partial p_E \partial p_S = \partial^2 \pi^E / \partial p_E \partial p_B = \partial^2 \pi^{PF} / \partial p_B \partial p_E = 0$, we have

$$\begin{aligned}
a_2 &= (\partial^2 \pi^{PF} / \partial p_S \partial p_B)(\partial^2 \pi^{PF} / \partial p_S \partial p_E), \\
a_3 &= -(\partial^2 \pi^{PF} / \partial p_S \partial p_B)(\partial^2 \pi^E / \partial p_E^2), \\
b_2 &= (\partial^2 \pi^{PF} / \partial p_B^2)(\partial^2 \pi^{PF} / \partial p_S^2) - (\partial^2 \pi^{PF} / \partial p_S \partial p_B)^2 = \Delta_0 > 0, \\
b_3 &= 0, \\
c_2 &= -(\partial^2 \pi^{PF} / \partial p_B^2)(\partial^2 \pi^{PF} / \partial p_E \partial p_S), \\
c_3 &= (\partial^2 \pi^{PF} / \partial p_B^2)(\partial^2 \pi^E / \partial p_E^2).
\end{aligned}$$

In the case of linear demands, we have

$$\begin{aligned}
\partial^2 \pi^{PF} / \partial p_B \partial a &= 0 > 0, \\
\partial^2 \pi^E / \partial p_E \partial a &= -(\partial D_B^E / \partial p_E) D_S(p_S) > 0, \\
\partial^2 \pi^{PF} / \partial p_S \partial a &= (\partial D_S / \partial p_S) D_B^E(p_E) < 0, \\
\partial^2 \pi^{PF} / \partial p_S \partial p_B|_{P^*} &= D_S(p_S)(\partial D_B / \partial p_B) < 0, \\
\partial^2 \pi^{PF} / \partial p_S \partial p_E &= (\partial D_B^E / \partial p_E) [D_S(p_S) + (\partial D_S / \partial p_S)(a + p_S - c)] \\
\partial^2 \pi^{PF} / \partial p_S^2 &= (\partial D_S / \partial p_S) D_B < 0, (*\text{Equation simplified under linear demands}) \\
\partial^2 \pi^{PF} / \partial p_B^2 &= 2(\partial D_B^{PF} / \partial p_B) D_S(p_S) < 0, (*\text{Equation simplified under linear demands}) \\
\partial^2 \pi^E / \partial p_E^2 &= 2(\partial D_B^E / \partial p_E) D_S(p_S) < 0,
\end{aligned}$$

We compute the determinant of the matrix

$$\begin{aligned}
\det M &= (\partial^2 \pi^{PF} / \partial p_B^2)(\partial^2 \pi^E / \partial p_E^2)(\partial^2 \pi^{PF} / \partial p_S^2) - \partial^2 \pi^{PF} / \partial p_S \partial p_B|_{P^*}^2 (\partial^2 \pi^E / \partial p_E^2) \\
&= (\partial^2 \pi^E / \partial p_E^2) \Delta_0 < 0.
\end{aligned}$$

Because of the equations above, we have $b_2 > 0$ and $\partial^2 \pi^{PF} / \partial a \partial p_E < 0$. Since $\det M < 0$, this implies that $dp_E^e / da > 0$. We have

$$dp_S^e / da = \frac{-2D_S^2(p_S)}{\det M} \frac{\partial D_B^{PF}}{\partial p_B} \frac{\partial D_B^E}{\partial p_E} \left[\frac{\partial D_B^E}{\partial p_E} D_S(p_S) + \frac{\partial D_S}{\partial p_S} \left(\frac{\partial D_B^E}{\partial p_E} (a + p_S - c) + 2D_B^E \right) \right].$$

If the sensitivity of seller demand is low, since $\partial D_B^{PF} / \partial p_B < 0$, $\partial D_B^E / \partial p_E < 0$, and $\det M < 0$,

we have that $dp_S^e/da < 0$. If the sensitivity of seller demand and the sensitivity of the entrant's demand are high, the price that the platform chooses for sellers is decreasing with a for low values of the access charge and then increasing with a for high values of the access charge.

Proof of Proposition 3: I start by proving the first part of Proposition 3. If the platform incurs losses on the entrant's transactions, the analysis of the direct and the indirect impact of the access charge on the platform's profits reveals that the platform's profits increase with the access charge. Consequently, if the platform accommodates entry, it chooses the highest access charge such that the entrant makes zero profit. This level of the access charge is denoted by \bar{a} . Therefore, the platform accommodates entry its profit if it sets \bar{a} is higher than its monopoly profit.

I now prove the second part of Proposition 3. The platform earns more profits when it accommodates entry if inequation (8) is satisfied. The first-order condition on the buyer side (which is the same if the entrant has entered and if the entrant has not entered) enables me to define p_B as a function of the seller price p_S , that I denote by $\widetilde{p}_B(p_S)$. I define $\widetilde{\pi}(p_S) = D_S(p_S)D_B(\widetilde{p}_B(p_S))(\widetilde{p}_B(p_S) + p_S - c)$. We have that $\widetilde{\pi}$ is decreasing in p_S . Inequation (8) can be rearranged as

$$\widetilde{\pi}(p_S^e) - \widetilde{\pi}(p_S^m) + D_B^E(p_E^e)(a + p_S^e - c) \geq 0.$$

If there exists $a \geq p_B^m$ such that $a + p_S^e - c \geq 0$, then, from Proposition 1, we have that $p_S^e \leq p_S^m$ and $p_B^e \geq p_B^m$. Since $\widetilde{\pi}$ is decreasing in p_S , this implies that $\widetilde{\pi}(p_S^e) - \widetilde{\pi}(p_S^m) \geq 0$. Since $a + p_S^e - c \geq 0$, this implies that inequation (8) is always true.

The socially optimal access charge: In this subsection, I assume that a regulator sets the access charge that maximizes the sum of consumer surplus, seller surplus, the platform's profits, and the entrant's profits. In our example with linear demands, we have

$$\begin{aligned} \frac{dS_B^{PF}}{da} &= -D_B^{PF}(p_B^e)D_S(p_S^e)\frac{dp_B^e}{da} + \frac{dp_S^e}{da}\frac{dD_S}{dp_S}\int_{p_B}^{\bar{b}_B}(b_B - p_B)h_B(b_B)db_B \\ &= -D_B^{PF}(p_B^e)D_S(p_S^e)\frac{dp_B^e}{da} - \frac{dp_S^e}{da}\frac{(1 - \gamma p_B)^2}{2}, \\ \frac{dS_B^E}{da} &= -D_B^E(p_E^e)D_S(p_S^e)\frac{dp_E^e}{da} + \frac{dp_S^e}{da}\frac{dD_S}{dp_S}\int_{p_E}^{\bar{b}_E}(b_E - p_E)h_E(b_E)db_E \\ &= -D_B^E(p_E^e)D_S(p_S^e)\frac{dp_E^e}{da} - \frac{dp_S^e}{da}\frac{(1 - \delta p_B)^2}{2}, \\ \frac{dS_S}{da} &= -(D_B^E(p_E^e) + D_B^{PF}(p_B^e))D_S(p_S^e)\frac{dp_S^e}{da} - \left(\gamma\frac{dp_B^e}{da} + \delta\frac{dp_E^e}{da}\right)\frac{(1 - \gamma p_S)^2}{2\gamma}. \end{aligned}$$

Best-response functions and strategic complementarity: From (11) and from the implicit function theorem we have that

$$\frac{\partial p_E^{BR}}{\partial p_B} = - \left(\frac{\partial^2 \pi^E}{\partial p_E^2} \right)^{-1} \frac{\partial^2 \pi^E}{\partial p_B \partial p_E}.$$

Since $\partial^2 \pi^E / \partial p_E^2 = 2(\partial D_B^E / \partial p_E) D_S(p_S) < 0$ and $\partial^2 \pi^E / \partial p_E \partial p_B = D_S(p_S)(\partial D_B^E / \partial p_B) > 0$, this implies that the entrant's best response is increasing with the price chosen by the platform on the buyers' side.

Differentiating (9) and (10) with respect to p_E , we obtain that

$$\begin{pmatrix} \partial p_B^{BR} / \partial p_E \\ \partial p_S^{BR} / \partial p_E \end{pmatrix} = -\frac{1}{\Delta_0} \begin{pmatrix} \partial^2 \pi^{PF} / \partial p_S^2 & -\partial^2 \pi^{PF} / \partial p_B \partial p_S \\ -\partial^2 \pi^{PF} / \partial p_B \partial p_S & \partial^2 \pi^{PF} / \partial p_B^2 \end{pmatrix} \begin{pmatrix} \partial^2 \pi^{PF} / \partial p_B \partial p_E \\ \partial^2 \pi^{PF} / \partial p_E \partial p_S \end{pmatrix},$$

where $\Delta_0 = (\partial^2 \pi^{PF} / \partial p_B^2)(\partial^2 \pi^{PF} / \partial p_S^2) - (\partial^2 \pi^{PF} / \partial p_S \partial p_B)^2 > 0$ by assumption of concavity of the platform's profits. This implies that

$$\frac{\partial p_B^{BR}}{\partial p_E} = -\frac{1}{\Delta_0} \left(\frac{\partial^2 \pi^{PF}}{\partial p_S^2} \frac{\partial^2 \pi^{PF}}{\partial p_B \partial p_E} - \frac{\partial^2 \pi^{PF}}{\partial p_B \partial p_S} \frac{\partial^2 \pi^{PF}}{\partial p_E \partial p_S} \right),$$

and

$$\frac{\partial p_S^{BR}}{\partial p_E} = -\frac{1}{\Delta_0} \left(\frac{\partial^2 \pi^{PF}}{\partial p_B^2} \frac{\partial^2 \pi^{PF}}{\partial p_S \partial p_E} - \frac{\partial^2 \pi^{PF}}{\partial p_B \partial p_S} \frac{\partial^2 \pi^{PF}}{\partial p_E \partial p_B} \right).$$

For the platform's best-response on the buyer side to be increasing in p_E , we have to assume that

$$\Delta_1 = \frac{\partial^2 \pi^{PF}}{\partial p_S^2} \frac{\partial^2 \pi^{PF}}{\partial p_B \partial p_E} - \frac{\partial^2 \pi^{PF}}{\partial p_B \partial p_S} \frac{\partial^2 \pi^{PF}}{\partial p_E \partial p_S} < 0.$$

Proof of Proposition 4: In our example, we have

$$p_B^e - p_S^e = \frac{36 - 3r + s(33 - 4r + s(r + (8 - s)(-4 + s)s) + 2a(-18 + s(11 + s(-5 + (-1 + s)s))))}{4s^2(8 + s - 4s^2 + s^2)},$$

where

$$\begin{aligned} r^2 = & (144 + s(-120 + 4a^2(-2 + s)s(-18 + s(13 + (-6 + s)s))) + s(89 + s(-64 + s(30 + (-8 + s)s))) \\ & + 4a(-2 + s)(36 + s(-19 + s(13 + (-7 + s)s)))). \end{aligned}$$

If the degree of product differentiation s is low (e.g., $b = 0.1$), we have $p_B^e - p_S^e \geq 0$. Otherwise, if the degree of product differentiation is high (e.g., $b = 0.9$), we have $p_B^e - p_S^e \leq 0$ for low values of a and $p_B^e - p_S^e \geq 0$ for high values of a .

We have

$$p_B^e - p_E^e = \frac{12 - r + s(7 - r + s(-5 + (-3 + s)s) + 2a(-6 - 5s + s^3))}{4s(8 + s - 4s^2 + s^2)} \leq 0.$$

Proof of Lemma 3: By differentiating the first-order conditions with respect to a , we obtain

that

$$\begin{pmatrix} -\partial^2 \pi^{PF} / \partial p_B \partial a \\ -\partial^2 \pi^E / \partial p_E \partial a \\ -\partial^2 \pi^{PF} / \partial p_S \partial a \end{pmatrix} = \begin{pmatrix} \partial^2 \pi^{PF} / \partial p_B^2 & \partial^2 \pi^{PF} / \partial p_B \partial p_E & \partial^2 \pi^{PF} / \partial p_B \partial p_S \\ \partial^2 \pi^E / \partial p_B \partial p_E & \partial^2 \pi^E / \partial p_E^2 & \partial^2 \pi^E / \partial p_E \partial p_S \\ \partial^2 \pi^{PF} / \partial p_B \partial p_S & \partial^2 \pi^{PF} / \partial p_E \partial p_S & \partial^2 \pi^{PF} / \partial p_S^2 \end{pmatrix} \begin{pmatrix} dp_B^e / da \\ dp_E^e / da \\ dp_S^e / da \end{pmatrix}.$$

Denoting by M the matrix of cross-derivatives, this implies that

$$\begin{pmatrix} dp_B^e / da \\ dp_E^e / da \\ dp_S^e / da \end{pmatrix} = M^{-1} \begin{pmatrix} -\partial^2 \pi^{PF} / \partial p_B \partial a \\ -\partial^2 \pi^E / \partial p_E \partial a \\ -\partial^2 \pi^{PF} / \partial p_S \partial a \end{pmatrix},$$

and therefore that

$$\begin{aligned} dp_B^e / da &= \frac{-1}{\det M} (a_1 \partial^2 \pi^{PF} / \partial p_B \partial a + a_2 \partial^2 \pi^E / \partial p_E \partial a + a_3 \partial^2 \pi^{PF} / \partial p_S \partial a), \\ dp_E^e / da &= \frac{-1}{\det M} (b_1 \partial^2 \pi^{PF} / \partial p_B \partial a + b_2 \partial^2 \pi^E / \partial p_E \partial a + b_3 \partial^2 \pi^{PF} / \partial p_S \partial a), \\ dp_S^e / da &= \frac{-1}{\det M} (c_1 \partial^2 \pi^{PF} / \partial p_B \partial a + c_2 \partial^2 \pi^E / \partial p_E \partial a + c_3 \partial^2 \pi^{PF} / \partial p_S \partial a). \end{aligned}$$

Since $\partial^2 \pi^E / \partial p_E \partial p_S = 0$, we have

$$\begin{aligned}
a_1 &= (\partial^2 \pi^E / \partial p_E^2) (\partial^2 \pi^{PF} / \partial p_S^2), \\
a_2 &= (\partial^2 \pi^{PF} / \partial p_S \partial p_B) (\partial^2 \pi^{PF} / \partial p_S \partial p_E) - (\partial^2 \pi^{PF} / \partial p_B \partial p_E) (\partial^2 \pi^{PF} / \partial p_S^2), \\
a_3 &= -(\partial^2 \pi^{PF} / \partial p_S \partial p_B) (\partial^2 \pi^E / \partial p_E^2), \\
b_1 &= -(\partial^2 \pi^E / \partial p_E \partial p_B) (\partial^2 \pi^{PF} / \partial p_S^2), \\
b_2 &= (\partial^2 \pi^{PF} / \partial p_B^2) (\partial^2 \pi^{PF} / \partial p_S^2) - (\partial^2 \pi^{PF} / \partial p_S \partial p_B)^2, \\
b_3 &= (\partial^2 \pi^{PF} / \partial p_S \partial p_B) (\partial^2 \pi^E / \partial p_E \partial p_B), \\
c_1 &= (\partial^2 \pi^E / \partial p_E \partial p_B) (\partial^2 \pi^{PF} / \partial p_E \partial p_S) - (\partial^2 \pi^E / \partial p_E^2) (\partial^2 \pi^{PF} / \partial p_S \partial p_B), \\
c_2 &= (\partial^2 \pi^{PF} / \partial p_B \partial p_E) (\partial^2 \pi^{PF} / \partial p_S \partial p_B) - (\partial^2 \pi^{PF} / \partial p_B^2) (\partial^2 \pi^{PF} / \partial p_E \partial p_S), \\
c_3 &= (\partial^2 \pi^{PF} / \partial p_B^2) (\partial^2 \pi^E / \partial p_E^2) - (\partial^2 \pi^{PF} / \partial p_B \partial p_E) (\partial^2 \pi^E / \partial p_B \partial p_E).
\end{aligned}$$

In the case of linear demands, we have

$$\begin{aligned}
\partial^2 \pi^{PF} / \partial p_B \partial a &= D_S(p_S) (\partial D_B^E / \partial p_B) > 0, \\
\partial^2 \pi^E / \partial p_E \partial a &= -D_S(p_S) (\partial D_B^E / \partial p_E) > 0, \\
\partial^2 \pi^{PF} / \partial p_S \partial a &= D_B^E(p_B, p_S) (\partial D_S / \partial p_S) < 0, \\
\partial^2 \pi^{PF} / \partial p_E \partial p_B &= -D_S(p_S) (\partial D_B^{PF} / \partial p_E) > 0, \\
\partial^2 \pi^{PF} / \partial p_S \partial p_B|_{P^*} &= D_S(p_S) (\partial D_B / \partial p_B) < 0, \\
\partial^2 \pi^{PF} / \partial p_S \partial p_E &= D_S(p_S) (\partial D_B / \partial p_E) + (\partial D_S / \partial p_S) ((\partial D_B^E / \partial p_E)(a + p_S - c) + (\partial D_B^{PF} / \partial p_E)(p_B + p_S - c)) \\
\partial^2 \pi^{PF} / \partial p_S^2 &= (\partial D_S / \partial p_S) D_B < 0, (*\text{Equation simplified under linear demands}) \\
\partial^2 \pi^{PF} / \partial p_B^2 &= 2(\partial D_B^{PF} / \partial p_B) D_S(p_S) < 0, (*\text{Equation simplified under linear demands}) \\
\partial^2 \pi^E / \partial p_E^2 &= 2(\partial D_B^E / \partial p_E) D_S(p_S) < 0, \\
\partial^2 \pi^E / \partial p_S \partial p_E &= 0, \\
\partial^2 \pi^E / \partial p_E \partial p_B &= D_S(p_S) (\partial D_B^E / \partial p_B) > 0.
\end{aligned}$$

We compute the determinant of the matrix

$$\begin{aligned}
\det M &= (\partial^2 \pi^{PF} / \partial p_B^2)(\partial^2 \pi^E / \partial p_E^2)(\partial^2 \pi^{PF} / \partial p_S^2) + \partial^2 \pi^{PF} / \partial p_S \partial p_B \big|_{P^*} (\partial^2 \pi^E / \partial p_E \partial p_B)(\partial^2 \pi^{PF} / \partial p_S \partial p_E) \\
&\quad - \partial^2 \pi^{PF} / \partial p_S \partial p_B \big|_{P^*}^2 (\partial^2 \pi^E / \partial p_E^2) - (\partial^2 \pi^{PF} / \partial p_S^2)(\partial^2 \pi^E / \partial p_E \partial p_B)^2 \\
&= (\partial^2 \pi^E / \partial p_E^2) \Delta_0 - (\partial^2 \pi^E / \partial p_E \partial p_B) \Delta_1 < 0.
\end{aligned}$$

Because of the equations above, we have that $a_1 > 0$, $\partial^2 \pi^{PF} / \partial p_S \partial p_B > 0$, $a_2 > 0$ (since prices are strategic complements), $\partial^2 \pi^E / \partial p_E \partial a > 0$, $a_3 < 0$ and $\partial^2 \pi^E / \partial p_S \partial a < 0$. Since $\det M < 0$, this implies that $dp_B^e / da > 0$. We have that $b_1 > 0$, $b_2 > 0$, $b_3 < 0$, $\partial^2 \pi^{PF} / \partial a \partial p_B > 0$, $\partial^2 \pi^E / \partial p_B \partial a > 0$, and $\partial^2 \pi^{PF} / \partial a \partial p_S < 0$. Since $\det M < 0$, this implies that $dp_E^e / da > 0$. Since the sign of $\partial^2 \pi^{PF} / \partial p_E \partial p_S$ is ambiguous, the sign of dp_S^e / da is ambiguous.